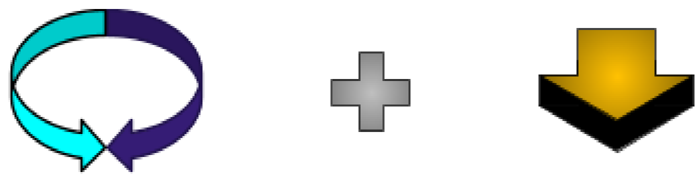

Superconducting Qubits and YOU!

a primer in superconducting quantum computing

and mediocre graphic design by Thomas N. Tsekerides

The development of quantum computers is one of the greatest engineering challenges of the 21st century. We as budding scientists have a duty to develop an understanding of this problem, no matter our eventual chosen discipline. This document serves as an effort to provide such an understanding of one particular solution to storing and manipulating quantum information, that is, by utilizing the properties of superconductive materials. Superconducting quantum bits exhibit quantum behavior on a macroscopic scale. They consist of circuits which act as highly controllable and predictable “artificial atoms” with simple, discrete energy levels that can be placed in superposition states. Unlike photonic or atomic qubits, superconducting qubits are readily scalable to the levels required to perform useful computations, as they can be produced by methods similar to those used in classical chip manufacturing.

This primer assumes a basic understanding of both classical and quantum physics and computing, but we will rehash some things for the sake of literary coherency.

What do we need to make a qubit?

Let's begin as many a primer in quantum computing has done before: "information is physical".[1,2] Whether it be chalk on a board, pixels on a screen, or waves of compressions in the air, something in the real world is there carrying the information. In computation, the basic unit of information is the bit, a binary logical state of zero or one. In a classical computer, a bit is stored as the presence (the one state) or absence (the zero state) of a few hundred thousand electrons on a capacitor in a Dynamic Random Access Memory (DRAM) circuit. Quantum physics tells us it is possible to create a quantum bit that is in a superposition of both the zero and one state, and that we can do that using the two lowest energy levels of a quantum mechanical object. As we all know, we can make a circuit mimic the motion of a particle by using an inductor and a capacitor to make an LC circuit. So, to make a qubit out of a circuit, all we need is to be able to make an LC resonator with a Hamiltonian which looks like a quantum harmonic oscillator and in which we can control the superposition state of the two lowest energy levels.

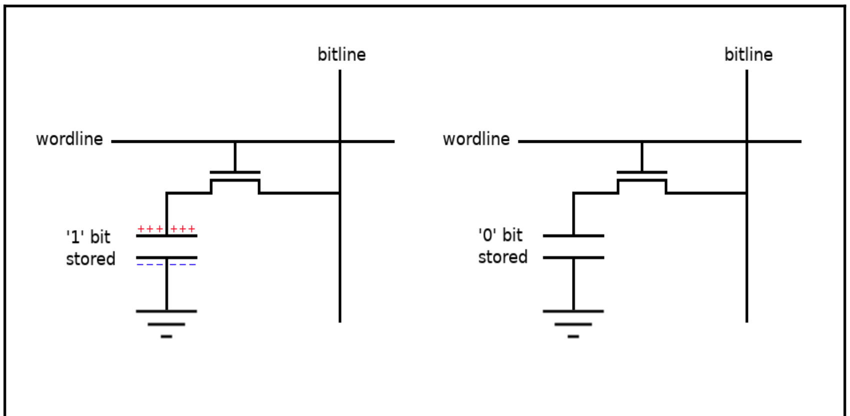

Figure 1: A DRAM circuit

Huzzah! Unfortunately, quantum physics also tells us it's going to be difficult to do this. Please, restrain your huzzahs. Qubits must be as isolated from the world as possible in order to achieve long coherence times, because the second something observes them they will collapse into a classical state. But we also need to be able to rapidly change the qubit's state and get accurate readouts when doing computations, both of which require strong coupling to exterior systems[4]. These two optimization parameters are clearly in direct conflict with each other.

So how can we manage these constraints and make our quantum-mechanical resonator circuit? Well, we'll need a little help from…

Superconductors

Superconductors are materials which when cooled to a critical point (usually near absolute zero), electrons of opposite spin will exchange phonons and form Cooper pairs, all of which move together as a unit with next to no resistance[2,3]. Cooling is needed because the thermal energy of the superconducting material needs to be low enough such that this coupling isn't broken the second it forms. We don't really need to get into the details of this phenomenon much beyond that, as I can say from firsthand experience that that will suck us down a deep rabbit hole.

The reason we use hundreds of thousands of electrons in a DRAM circuit is that they have a tendency to leak electrons out into the world, flipping bits and producing errors. But if we try to resist noise problems in a qubit by making it larger, we increase the likelihood of errors occurring due to interaction with the outside world. As current moves in normal metals, things are constantly bumping into each other, destroying superpositions[2]. But by building our circuits with superconducting metal instead, we eliminate this problem.

Now we can start to observe and utilize quantum mechanical behavior on a macroscopic scale. Much like how a crystal's macroscopic structure arises from its atomic level structure, quantum mechanical behavior of Cooper pairs in a superconducting circuit leads to situations where we can observe macroscopic quantum behavior. So let's build a superconducting LC circuit! The Hamiltonian of an LC circuit is $H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$. In the mechanical oscillator analogy, that last equality is taking the position coordinate taken to be φ, the "gauge invariant phase" across the inductor, and the conjugate momentum to be Q, the charge on the capacitor[3,5]. The mass analogue is then C and the spring constant is 1/L. We can make this quantum mechanical by using the commutation relation $[\hat{\Phi}, \hat{Q}] = \hat{\Phi}\hat{Q} \quad \hat{Q}\hat{\Phi} = i$. This leaves us with the Hamiltonian $H = 4E_C n^2 + \frac{1}{2}E_L \varphi^2$. Great! Now this looks like the potential well of a particle, where φ is the position. The excitation of the electrons is now described by a wave function dependent on φ, just like an atom! Let's look at a graph of the potential (Figure 2).

Uh oh. We have an issue. Recall, we need to be able to limit ourselves to the two lowest eigenstates. In order to do this, we need anharmonicity between the energy levels, that is, we need the frequencies of the transitions between them to be as different as possible, so we can maximize our control over the states. Remember, we want to limit ourselves to exciting the two lowest levels. The more different the frequencies which excite transitions are, the easier it is for us to apply a pulse of a frequency that will only excite changes between the lowest two states, and not any higher ones[5].

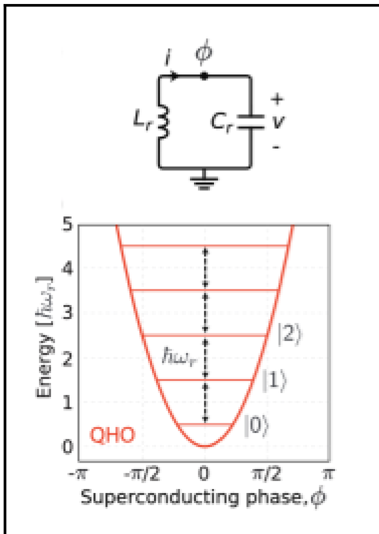

Figure 2: Circuit diagram and plot of the Hamiltonian potential of an LC resonator

How do we get this? Look at the Hamiltonian. Clearly, the φ term is what produces this parabolic curve. We want that term to become nonlinear. To get that, we introduce a special electrical component which relies on superconductivity and the properties of quantum mechanics to produce a nonlinear inductance.

Josephson Junction: What's Your Function?

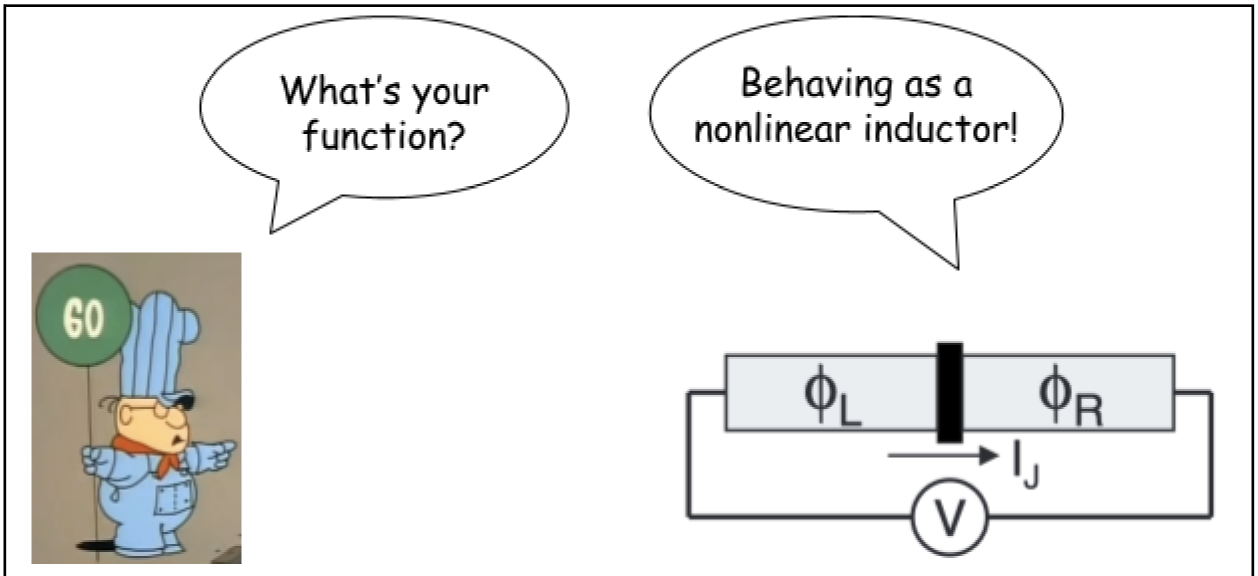

Figure 3: Interrogation of a Josephson junction.

A key component to building a superconducting qubit is the Josephson Junction, a nonlinear, nondisipative circuit element consisting of two superconductors with a thin layer of insulating material in between then, through which electrons (coupled in Cooper Pairs) can tunnel in a phenomenon called the Josephson effect[3,5]. Current flows through the junction according to the classical equations $I_J = I_0 \sin(\delta)$ and $V = \frac{\Phi o}{2\pi}\frac{d\delta}{dt}$, where $\Phi_0 = h/2e$ is the superconducting flux quantum (2e, because the charge carriers are Cooper pairs), I_0 is the critical current of the junction, and δ is the phase difference across the junction[4,5]. With a little differentiation and defining the Josephson induction by using the conventional $V = L_J \frac{dI_J}{dt}$, we can see that the Josephson Junction behaves as a nonlinear inductor, with its Josephson inductance described by the equation $L_J = \frac{\Phi o}{2\pi I_0 cos(\delta)}$ [4]. This gives rise to a new Hamiltonian $H = 4E_C n^2 - E_J cos(\varphi)$, where $E_C = \frac{e^2}{2(C_J + C_s)}$ (C_s being the shunt capacitance and C_J the self-capacitance of the Josephson junction) and $E_J = \frac{I_C \Phi_0}{2\pi}$ is the Josephson energy[5]. With this change, the dynamics of the system is now governed by the ratio of E_J to E_C. As depicted in Figures 4 and 5, the potential has a sinusoidal shape, and thus anharmonicity between the energy levels. We can now limit ourselves to the computational subspace of the lowest two energy levels. There we go! Nonlinearity introduced. Qubit complete?

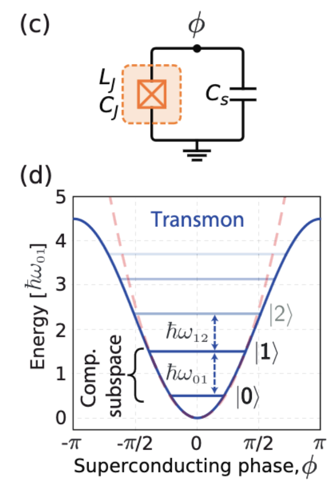

Figure 4: The circuit diagram and plot of the Hamiltonian potential of a Josephson junction - Capacitor resonator

Well… not really. Yes, our energy spectrum is no longer degenerate[4]. But we have some other problems to contend with. Let's examine some superconducting qubit circuit architecture, to see what these are and how we can contend with them.

Types of Superconducting Qubits: Charge, Phase, and Flux

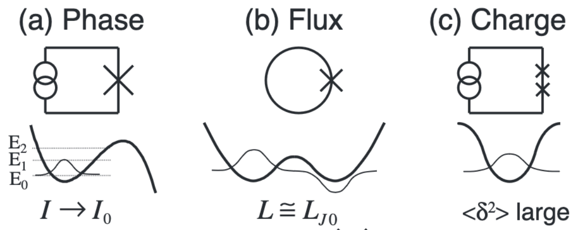

Figure 5: The various qubit architectures, with their circuit diagrams, plots of their Hamiltonians (bold line) and ground state wavefunctions (thin line)

There are three main types of superconducting qubit: the charge qubit, the phase qubit, and the flux qubit.

One of the oldest and simplest forms of superconducting qubit is a charge qubit called the Cooper pair box. This circuit is basically the one we've been looking at, if not even simpler, consisting of only an electrode with a tiny capacitance connected to a reservoir by a Josephson junction[6] . The capacitance of the electrode is small enough that adding a single Cooper pair takes a lot of energy[5]. The amount of energy required can be controlled with an adjustable bias voltage from the pulse gate. At a special bias voltage, we can create a superposition of there being a lot of Cooper pairs (~ a billion) or a lot plus one Cooper pairs on the electrode. This is similar to a DRAM circuit from earlier: information is stored in the form of charge on a capacitor, except here the charge difference is one Cooper pair tunneling coherently back and forth, connecting the energy levels of the a lot and a lot plus one states.

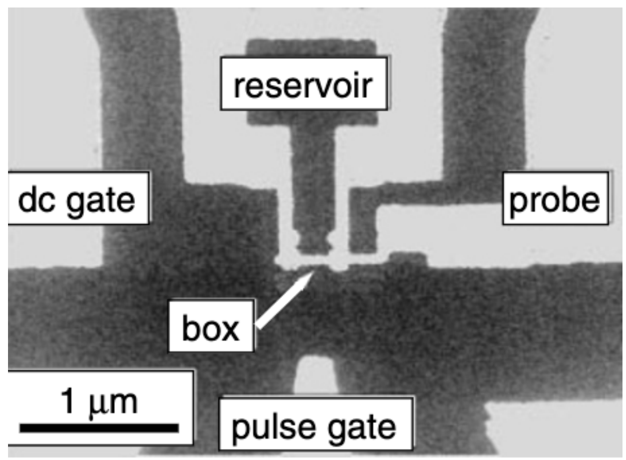

Figure 6: A Cooper pair box

The Cooper pair box may be the first superconducting qubit, but it's also the worst. In this design, $E_J \leq E_C$, making these qubits extremely susceptible to noise, and giving them coherence times of only a few nanoseconds[5]. A variation on the Cooper pair box called the transmon (depicted in Figure 4) attempts to deal with these problems by attaching a large shunting capacitance to the circuit, making $E_J >> E_C$. This makes the circuit much less susceptible to noise, but moderately reduces the anharmonicity between energy levels [2,5]. The transmon behaves like an atom with atomic number of 10^{12}, but its energy spectrum is simple at low energies. Due to its size, it has a huge dipole moment, and thus couples strongly with microwave signals for good control and readout[2]. If we add one extra junction, we'd still have a sinusoidal potential (we can eliminate one degree of freedom with something called the "fluxoid quantization condition")[5], but now one in which we can control the effective E_J with an external flux. This is called a split transmon. Unfortunately, the limited anharmonicity of the sinusoidal potential means that significant unwanted excitation of higher energy states can occur, putting a decent dent in the performance of gate operations[5]. But what if we could make that potential… wigglier? With the flux qubit, we can! By throwing an extra Josephson junction into the circuit loop, we can make a circuit that looks like Figure 7. On the left, we have a Josephson junction, on the right, two identical junctions, larger than the left by a factor of γ. By adding one more Josephson junction to the loop, we create the highly nonlinear curve shown in Figure 5b. The wonky potential curve is a result of adding quadratic potential of the shunt inductance with the Josephson junction's cosine potential. In this design, the inductor makes it so that the charge variable is continuous (not integer valued, as in the charge qubit)[3]. Rather than having a superposition of one Cooper pair existing between two sides of a tunnel junction, we have a superposition of clockwise and counterclockwise current. For this reason, it is also known as a Persisting Current Flux Qubit[5]. These qubits have great anharmonicity, on the order of several GHz, much higher than that of a transmon, with only

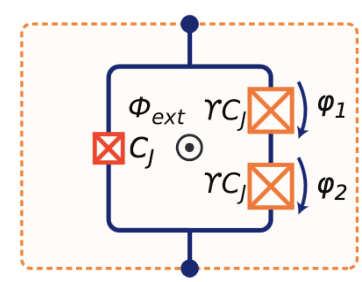

Figure 7: A flux qubit

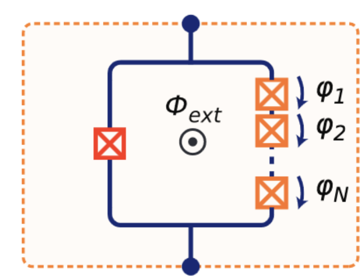

Figure 8: Fluxonium

~200MHz. Additionally, the transition frequencies between their energy levels (100 MHz - 1 GHz) are much lower than those in transmons (~5GHz), making them less susceptible to dielectric loss and improving coherence times[7].

An evolution of the flux qubit is fluxonium, a flux qubit with potentially hundreds of additional Josephson junctions which approximate a very large shunt inductor. This gives fluxonium very high anharmonicity and resistance to external flux[3]. Fluxonium has exhibited coherence times of more than a millisecond, and we'll discuss more work related to it in the next section.

The final flavor of superconducting qubit is the phase qubit. This is another inductively shunted design, set up such that transitioning between the ground and first excited state is difficult, but tunneling out of the third level becomes easier. This allows us to manipulate the qubit in the register of the lower two levels, apply pulse to readout, and, if it's in the excited state, get a big voltage spike. This allows for very strong readout, but destroys coherency of nearby qubits, making this design difficult to scale for useful computations[2].

Coupling Qubits and Fluxonium-Transmon-Fluxonium Coupling

Most useful quantum computations require at least two qubits with entanglement between them. With superconducting qubits, we usually create interactions between qubits in the form of electric and/or magnetic fields[5]. The hookup itself can be created using capacitors or inductors, although most two qubit gates rely on capacitive coupling[7]. Coupling is a critical part of turning individual qubits into quantum computers, but it's also where a lot of challenges arise: as we've learned, stronger coupling often means more chance for quantum errors!

Although fluxonium has appealing advantages over the transmon design, almost all advances in scaling up superconducting quantum processors have been with transmons. The smaller transition elements which give fluxonium such long coherence times also make its coupling weaker than that of a transmon. Additionally, capacitive coupling leads to an always on entangling rate (ZZ) – a type of constant error arising from higher transition states of qubits acting on the computational states.[7]

A quite recent development in coupling methodology is the fluxonium-transmon-fluxonium circuit, where two fluxonium qubits are coupled using a transmon. Strong capacitive couplings hybridize the first higher energy transition (1 → 2) of the fluxonium qubits with the lower transition (0 → 1) of the transmon, but the computational states themselves remain relatively separated. Thus, the qubits can perform well as both single and two qubit gates, with average CZ gate fidelities of 99.922 ± 0.009%. This coupling also reduces ZZ to kHz levels, and the system has exhibited coherence times of up to a millisecond[7].

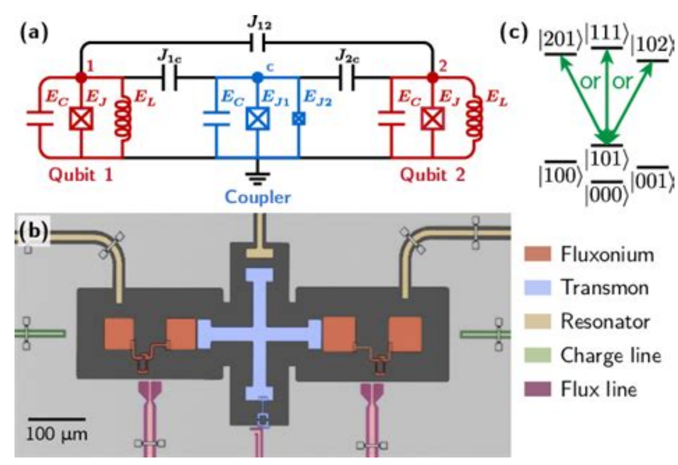

Figure 9: Fluxonium-Transmon-Fluxonium diagram. c) illustrates the function of the CZ gate.

This design opens the door to scaling up fluxonium-based quantum devices, and thus to even more innovation in super- conducting quantum computing.

Superconducting qubits — and YOU!

I hope you are now convinced that superconducting circuits are an extremely promising method for creating quantum computers. Although they exhibit lower coherence times than other methods, they are catching up, and the low dissipation of their materials means that long coherence times are possible[2]. They have great readout and controllability due to the strong coupling capabilities granted by their macroscopic size, and researchers are making big progress towards producing bigger and better quantum devices based on them. There is still a ***long*** way to go before we'll be able to make something really useful, and there are problems we haven't even discussed yet in this primer (for example, tiny manufacturing imperfections in the electronics can contribute significantly to errors)[1].

This is where you, the peers and friends for whom I am writing this, come in. This field is so young, and we are in a historical position where we may very well be the generation to take this technology from cutting-edge research to working machines. You! Yes, you, dear reader, could be a part of this effort. And even if not (I for one am more interested in fusion energy technologies), the technology is worth understanding. As Schoolhouse Rock so eloquently put it: "It's great to learn, cause knowledge is power".

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Images:

Figure 1 - St. Michael, S. (n.d.). *Introduction to DRAM (dynamic random-access memory) - technical articles*. All About Circuits. https://www.allaboutcircuits.com/technical-articles/introduction-to-dram-dynamic-random-access-memory/

Figure 2 - Krantz et al.

Figure 3 - Conductor :Schoolhouse Rock - “Conjunction Junction” Josephson Junction: Martinis

Figure 4 - Krantz et al.

Figure 5 - Martinis

Figure 6 - Nakamura, Y., Pashkin, Y. A., & Tsai, J. S. (1999). Coherent control of macroscopic quantum states in a single-Cooper-pair box. *nature*, *398*(6730), 786-788.

Figure 7 and 8 - Krantz et al.

Figure 9 - Ding et al.

Curiously quantum-y fortune from my dinner.